

# Pentaquark as Kaon–Nucleon Resonance

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Several recent experiments have reported evidence for a narrow feature in the  $K^+$ -neutron system, an apparent resonant state  $\sim 100$  MeV above threshold and with a width  $\leq 25$  MeV. This state has been labelled as  $\Theta^+$  (previously as  $Z^*$ ), and because of the implied inclusion of a anti-strange quark, is referred to as a pentaquark, that is, five quarks within a single bag. We present an alternative explanation for such a structure, as a higher angular momentum resonance in the isospin zero  $K^+N$  system. One might call this an exit channel or a molecular resonance. In a non-relativistic potential model we find a possible candidate for the kaon-nucleon system with relative angular momentum  $L = 3$ , while  $L = 1$  and 2 states possess centrifugal barriers too low to confine the kaon and nucleon in a narrow state at an energy so high above threshold. A rather strong state-dependence in the potential is essential, however, for eliminating an observable  $L = 2$  resonance at lower energies.

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Several groups [1, 2, 3, 4] have reported the presence of a resonance, referred to presently as the  $\Theta^+$ , in a variety of experimental configurations. This experimental situation stands in sharp contrast to that relative to the 6-quark H-dibaryon [5], which at first glance seemed theoretically to be a candidate for an exotic state eminently more likely to be discovered. For the  $H$  there have been many, so far fruitless searches. We offer elaborations on some points made by Capstick, *et al.* [6], on the general nature of resonances in the kaon-nucleon system and the connection between their angular momenta, widths and energies above threshold.

Perhaps the simplest data comes from the SAPHIR detector at ELSA [4], wherein the positive strangeness  $\Theta^+$  is photo-produced off a simple proton target. The final state contains  $nK^+K_s^0$  and the relevant system is identified in the missing mass spectrum of the  $K_s^0$ . The  $K_s^0$  is reconstructed from its two  $\pi$  decay, preferentially in the forward direction. These authors conclude the  $\Theta^+$  is an isoscalar due to the absence of a  $\Theta^{++}$  in the  $\gamma p \rightarrow pK^+K^-$  channel. Interestingly, in this preferentially forward detection of the  $K_s^0$  [4], the transfer of appreciable angular momentum to the observed resonance would be favored.

Previously and presently, several theorists [7, 8, 9, 10, 11] have predicted the existence of such states, on occasion with astonishing accuracy [9] one might add, using the Skyrme model topological soliton [12] as a basis; others, including the experimental groups themselves, [1, 2, 3, 6, 13] have described the state as a bag [14] containing five quarks, *viz.* a pentaquark.

There are perhaps some difficulties inherent in either of these theoretical approaches, since a  $^1P$  configuration is generally selected as the primary candidate. There is a wealth of low energy  $KN$  scattering data [15, 16, 17, 18, 20] which presumably would have revealed a low-angular momentum  $\Theta^+$  resonance in the total cross-section or elsewhere. This problem could be avoided if the state possessed a truly narrow width, perhaps  $\Gamma \leq 3$  MeV [21]. On the other hand if a higher angular momentum  $L =$

2, 3 resonance were sought, the absence of observations in  $\sim 100$  MeV scattering data might perhaps be bypassed [21].

Pentaquark bag and soliton solutions both face a related problem. They will, in decay, be connected to the outgoing state via a low angular momentum, molecular ‘doorway’ state [6]. At 100 MeV of excitation, an  $L = 1$  resonance could acquire so large a width as to be non-existent in the absence of  $SU_f(3)$  symmetry breaking, or, in nuclear structure terms the presence of a small ‘spectroscopic’ factor to suppress the decay [6, 22].

In searching for a suitable phenomenological ‘effective’  $K-N$  potential one finds that a resonance of width 7–21 MeV at the rather elevated excitation energy of 100 MeV can indeed be constructed. Such an effective potential presumably represents a final state interaction between the kaon and nucleon components, i. e. a molecule like doorway state. Proceeding in this fashion may amount to trading one problem for another, and involves an appreciable degree of artificiality. But in doing so we can throw some light on the experimental problems arising from the assumption that the  $\Theta^+$  possesses a low angular momentum.

It is in fact already known, generally from phase shift analyses of data [16, 17, 18], that the  $KN$  system possesses a considerable state dependence in its various  $S$  and  $P$  configurations. In particular the isoscalar singlet  $P$  seems to evidence the greatest attraction and in some modeling, the phase shift passes upwards through  $\pi/2$ , however, at the rather high relative momentum of 800 MeV/c [18, 19]. In this very brief contribution we only attempt to delineate the minimum requirements for a molecular resonance to exist,  $K^+ + N$ , rather than a single bag.

A possible candidate state with an appropriate and natural single particle width-energy relation will, as we shall see, possesses  $L = 3$ . If there is no dependence of the effective potential on orbital angular momentum, it is likely the  $L = 2$  channel also has a resonant state, which will generally be narrower and closer to threshold.

Thus an  $L = 2$  resonance remains a possibility, with perhaps a reduced cross-section for production considering somewhat large momentum transfer required.

In this picture, we have in mind, first the creation of a very short-lived seven-quark 'bag' and soon after the fission into the final  $\Theta^+$  and  $K_s^0$ . Both the initial object and the  $\Theta^+$  may well be deformed [14, 23]. The  $\Theta^+$ -molecule is likely prolate before its subsequent fission (decay) into its  $K$  and  $N$  components. We envision that the interaction between these components is mainly on their surface, perhaps a particle-vibrational coupling.

Explicit calculations were done using the non-relativistic code GAMOW due to T. Verse, K. F. Pal and Z. Balogh [24], in its original form and also in a version modified to essentially exclude interactions inside some radius, somewhat inside the 0.8–1 fm expected for the size of a nucleon bag. Non-relativistic kinematics should be adequate for  $K$ - $N$  system at the rather low relative energy of 100 MeV ascribed to the  $\Theta^+$ . From the outset, it is abundantly clear that a state of such energy and width cannot be easily sustained by the centrifugal barrier in states with  $L \leq 2$ . This could be an advantageous circumstance, should further experiments truly eliminate the presence of even a narrow  $\Theta^+$  in low energy  $KN$  scattering. An  $L = 3$  resonance would have been very difficult to see in existing 100 MeV data [21]. The widths considered here are essentially what is referred to as single-particle in nature and ignore for now narrowing arising from any symmetry-breaking or other mechanism.

One potential which can produce a resonance in the higher partial waves is a surface Saxon-Woods form:

$$V_{surface}(r) = -V_0^s \left[ \frac{4e^{(r-R)/a}}{(1 + e^{(r-R)/a})^2} \right]. \quad (1)$$

The selection of a surface interaction, sharply peaked at that, is discussed below. If one wants to limit the produced resonances to say  $L = 2, 3, 4$ , some appreciable degree of state dependence must be introduced. One would also want to cut off this dependence in even higher partial waves. The potential strength then appears as,

$$V_0^s = A[\alpha + \beta F(L)], \quad (2)$$

There is no problem introducing *some* state dependence, certainly for isospin; most earlier treatments [16, 17] present the  $S$  and  $P$  states quite differently. More interestingly, Hasenfratz and Kuti [14, 25, 26, 27] suggested that isolated quark bags may be treated like liquid drops, in close analogy with nuclei. In such a dynamical treatment, the bag is deformable, and susceptible to surface oscillations which can be expanded in spherical harmonics, each characterized by a quantum number  $l$ . One can imagine, as one example, a particle-vibrational coupling producing the effective interaction in Eq.(1), by a quark in say the kaon coupling to a vibration of the

nucleon surface, thus our choice of  $R$ . These latter authors [25] point out that the surface potential energy is proportional to

$$c_l = (l-1)(l+2)\rho_0^2\sigma, \quad (3)$$

where  $\rho_0^2$  is the bag radius and  $\sigma$  the surface tension. Such a coupling might then lead to a comparable dependence in the overall effective  $K$ - $N$  interaction via particle-vibrational coupling. The number of surface waves contributing is naturally cut off at higher  $l$  by the the underlying microscopic structure of the bag surface, thus only a very few surface waves need enter. From Eq(3) the  $l = 1$  mode is absent and it is not unreasonable, given that only five valence quarks are present [27], that we assume only relative D, F and G waves are affected. Even an  $l = 4$  surface wave may have too small a wave length for consideration given that three valence quarks are present in the nucleon.

The particle-vibrational coupling in nuclei is, in any case certainly strongly surface peaked, with a form factor  $\sim rdV^{vol}/dr$  [27]. One then expects the diffusivity  $a$  to be quite small. We might assume for the surface strength a form like:

$$V_0^S = A[\alpha + \beta(L-1)(L+n)], \quad (4)$$

which is, of course, not directly justified and should be viewed as a phenomenological ansatz. Eq(4) implies no effect of the particle-vibrational coupling in the  $L = 1$  system and adds some further orbital dependence. Since we are considering only  $L = 2, 3, 4$  states here, we could simply quote results for differing ratios of potential strengths in these orbits, ignoring in effect the explicit choices made in this equation.

One can argue further that the simple kaon-nucleon picture should be modified to account for a bag-like interior, say within separations  $\sim 1.0$  fm, with the molecular state arising from interactions at larger distance. So a better model would be hybrid in nature, its states being mixtures of an interior quark bag and exterior hadrons. The surface potential employed here, with a small diffusivity, achieves this to some extent, but perhaps the interior structure is more complicated. A rather obvious, but nevertheless arbitrary, change would be to introduce a constant volume potential fixed to cancel the centrifugal force just inside the assumed surface and thus achieve some average cancellation in the interior, or alternatively to just eliminate this latter interaction inside. The first procedure reduces the required strength of the surface interaction  $V_0^S$ ; the second narrows the relevant  $L$ -state resonances considerably.

The radius parameter in Eq(1) is taken as  $R = 1.1(1.0087)^{1/3}fm = 1.130$  fm and the diffusivity as  $a = 0.2$  fm. The narrowness of the surface well, i. e. the small value of  $a$ , implies that the interaction is not strong inside or outside the chosen surface separation. The choice of

diffusivity greatly influences the resonance width;  $a$  and  $\Gamma$  both decrease together.

Specifically, using a potential with no interior centrifugal force and setting  $\alpha = 0$ ,  $n = 0$ ,  $\beta = 1$  and  $A = 70.835$  MeV yields an  $L = 3$ ,  $KN$  state at  $\epsilon = 103 - 3.76i$  MeV. For an unaltered interior potential with  $A = 134.2$  MeV, the  $L = 3$  state appears at  $\epsilon = 100 - 11.2i$  MeV. The widths  $\Gamma$  are twice the imaginary part of the energies. In all cases the radial integral of the surface potential, absent the angular factor  $4\pi$ , is near to or less than 50 MeV-fm<sup>3</sup>. Moreover this applies to only a few partial waves. The usually anticipated weakness of the  $K^+-N$  interaction [16] to some extent justifies setting  $\alpha = 0$ , or small, in Eq(4).

For  $\alpha = n = 0$  in Eq(4), the  $L = 2$  strength is reduced by  $1/3$ . With any of the models, even for less drastic reductions in the  $L = 2$  strength, this implies either that no  $D$ -state resonance exists, or that the state is present at some 84 MeV (no interior centrifugal force) but with a width  $\Gamma \sim 80$  MeV. On the other hand, if the surface potential strength is kept at the value for  $L = 3$  of  $6(70.83) = 425$  MeV, then a resonance develops in  $L = 2$  at  $\epsilon = 35.4 - 4.5i$  MeV. Such a  $D$  state, closer to threshold, is perhaps lower in production cross-section and more difficult to detect. Here we keep in mind the somewhat large momentum transfer occasioned by the production of at least one  $K$  meson in the final state.

In all situations the above  $L = 3$  surface potential alone is too weak to generate an  $L = 4$  resonance near 100 MeV. We expect in any case some cutoff effect to start with higher partial waves, probably by  $L = 4$ , since the nucleon and kaon possess few valence quarks. Similar results could have been obtained using energy-dependent potentials, but as we pointed out there is some justification for a particle-vibrational coupling acting in a few surface states.

This exercise certainly has artificial aspects, notably the state-dependent potentials, although such dependences are just what one would expect to arise in coupling the meson quarks to the nucleon bag surface. At first sight the surface potential depths appear large, however, since the diffusivity is small, the integrated moment of the potential, which has a direct significance in producing states is in fact quite small. One might well ask why such dynamics are limited to  $K^+-N$  and not also present in  $K^-N$ . Two answers are possible: the latter state is not exotic, proceeds through  $K^-$  absorption involving known and observed  $Y^*$  resonances in the  $s$ -channel, also, the  $K^+N$  system generally involves weaker interactions and may then more easily exhibit the surface effects discussed above.

The approach also has its advantages: the apparent experimental absence of the coupling of an assumed low angular momentum state to low energy  $K-N$  channels,

and the natural appearance at  $\sim 100$  MeV of an  $L = 3$  state with about the right width. Should closer experimental study reveal another, say  $L = 2$  resonance closer to threshold, one would have to take our approach more seriously. If the actual width of the  $\Theta^+$  proved to be truly small  $\leq 1$  MeV, then an explanation must probably be sought in some other model.

Finally one should note the rather large cross-section, found by the SAFIR collaboration [4],  $\sim 300$  nb, for production of the  $\Theta^+$ . As we noted above, the large change in mass, i. e. the production of one or two  $K$ 's in the final state, and the forward detection of the  $K_s^0$ , likely also favor an appreciable transfer of angular momentum to the putative  $\Theta$  resonance. This is especially clear if in  $\gamma p$ , the production of the final state proceeds through a doorway  $N^*$ . The nucleon form factor then enters the  $\gamma p$  cross-section roughly as  $|f_N[R_p 2M(K)]|^2$ , while if the  $K_s^0$  is produced at the first  $\gamma p$  vertex then perhaps only a single unit of  $M(K)$  is present. Such effects are seen, for example, in  $(\pi^+, K^+)$  reactions on nuclei, which preferentially produce high angular momentum hypernuclear levels [28].

To repeat: one lesson to be learned from this examination of an effective potential model, concerns the relationship between resonance energy and single particle width. It is difficult to obtain a narrow  $P$ -state at 100 MeV without symmetry breaking or other weakening of the decay. Capstick et. al. have already pointed this out and one can point to the strong but broad  $L = 1$ ,  $\Delta$  as a further example of it. The centrifugal force  $\sim 1/\mu$  sets the scale in any given system,  $\pi-N$  naturally having a higher energy scale than  $K-N$ . Nussinov et al. [22] discuss alternate means for narrowing the  $\Theta^+$  and related resonances. Should the as yet only weakly constrained width of the  $\Theta^+$ , at  $\Gamma \leq 25$  MeV, actually prove to be considerably narrower Refs. [6, 22] become even more relevant.

Capstick et al. have cleverly narrowed the width of the  $\Theta^+$  by taking this object to be an isotensor. They are then faced with the existence of the other members of this multiplet, in particular the  $\Theta^{++}$ . Our suggestion is clearly highly phenomenological, and a purely final-state kaon-nucleon potential model cannot be the whole answer, since very short distances must be described by the underlying quark nature of hadrons. Nevertheless, the relevance of somewhat higher orbital states cannot be eliminated immediately. Clearly, further experimental study of these exotic objects, in particular of their angular momenta, and of the entire low energy  $K-N$  system, is both interesting and necessary.

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